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AUTHOR	Crawford, Glenda
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ABSTRACT

The sixth in a series of six guidebooks on minimum course content for second-year algebra, this booklet presents an introduction to sequences, series, permutation, combinations, and probability. Included are arithmetic and geometric progressions and problems solved by counting and factorials. Overall course goals are specified, a course outline is provided, performance objectives are listed, and text references keyed to the performance objectives are included. Pre- and posttests are also given, together with answer keys. (JP)

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AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

MATHEMATICS: Algebra 2u 5216.26

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION•1973

QUINMESTER MATHEMATICS
COURSE OF STUDY
FOR

ALGEBRA 2u

5216.26

(EXPERIMENTAL)

Written by

Glenda Crawford

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
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Dr. E. L. Whigham, Superintendent of Schools
Dade County Public Schools
Miami, Florida 33132

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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

An introduction to sequences, series, permutations, combinations, and probability. Includes arithmetic and geometric progressions, problems solved by counting, and factorials.

Designed for the student who has mastered the skills and concepts of Algebra 2s.

TABLE OF CONTENTS

OVERALL GOALS

The senior high mathematics program provides experiences which enable each student, commensurate with his mathematical maturity and aptitude to:

a. Achieve competence in the basic arithmetic skills, gain understandings requisite for solving computational problems, and use the properties of mathematical structure.

b. Develop reading skills used in mathematics.

c. Develop the individual's ability to define, categorize, analyze, evaluate, interpret, and communicate through symbolic mathematical expressions in problem solving situations.

d. Appreciate the significant role of mathematics in the development of civilization in the past, present, and future, and become more aware of the ever increasing dependence that man has upon mathematics for his future development.

e. Develop both inductive and deductive reasoning in a mathematical context, with emphasis placed on their application to mathematical proofs and life situations.

Note: These overall goals come from Florida Standards 1971-72.

Goals: To develop those comprehensions and skills in the language of mathematics which will allow for further study in mathematics and science.

TEXT BIBLIOGRAPHY

D_3 - Dolciani, Mary P.; Berman, Simon L.; and Wooten, William. Modern Algebra and Trigonometry, Book 2. Boston: Houghton Mifflin Company, 1963.

D_8 - Dolciani, Mary; Wooten, William; Beckenbach, Edwin; Sharron, Sidney. Modern School Mathematics, Algebra II and Trigonometry. Boston: Houghton Mifflin Company, 1968.

N - Nichols, Eugene D.; Heimer, Ralph T.; Garland, Henry C. Modern Intermediate Algebra. New York: Holt Rinehart and Winston, Inc., 1965.

PL - Payne, Joseph N.; Zamboni, Floyd F.; and Lankford, Francis G., Jr. Algebra Two with Trigonometry. New York: Harcourt, Brace and World, Inc., 1969.

PA - Pearson, Helen R. and Allen, Frank B. Modern Algebra, A Logical Approach, Book Two. Boston: Ginn and Company, 1966.

OUTLINE

I. Sequence

1. Term of a sequence
2. Arithmetic sequence
3. $t_n = a + (n - 1)\,d$
4. Arithmetic means
5. Average

II. Series

1. $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$
2. Summation notation
3. Geometric sequence
4. $t_n = ar^{n-1}$
5. Geometric means
6. Geometric series
7. Infinite geometric series

III. Permutations

1. Linear permutations
2. Circular permutations
3. ${}_nP_r = n(n-1)\,(n-2)\,\ldots\,\left[n-(r-1)\right]$
4. ${}_nP_n = n!$
5. $\dfrac{n!}{n_1!\;n_2!\;\ldots}$

IV. Combinations

1. ${}_nC_r = \dfrac{{}_nP_r}{r!}$
2. ${}_nC_r = \dfrac{n!}{r!\,(n-r)!}$
3. ${}_nC_r = {}_nC_{n-r}$

V. Probability

1. Same prespace
2. Event
3. Evaluating probabilities

PERFORMANCE OBJECTIVES

The student will:

1. Define a sequence.
2. Identify an arithmetic progression.
3. Determine the missing term in $t_n = a + (n-1)d$, given the necessary information.
4. Insert a given number of arithmetic means between two real numbers.
5. Define a series.
6. Find the sum of an arithmetic progression given the necessary data.
7. Find the sum of an arithmetic series when the series is written with the summation sign.
8. Define a geometric progression.
9. Determine the missing term in $t_n = ar^{n-1}$, given the necessary information.
10. Insert a given number of geometric means between any two non-successive terms of a geometric series.
11. Define a geometric series.
12. Find the sum of any missing values of a finite geometric series given the necessary data.
13. Change repeating decimals to equivalent common fractions using the formula
$$S = \frac{a}{1-r}.$$
14. Define a permutation.
15. Find the number of permutations of a set containing n different elements.
16. Find the number of circular permutations of a set of n objects.
17. Evaluate n^Pr.
18. Evaluate n^Pn.
19. Find the number of permutations of n different elements taken r at a time.

20. Find the number of permutations of n elements taken n at a time with p elements alike, q elements alike, r elements alike and so on.

21. Define a combination.

22. Evaluate $\frac{{}_nP_r}{r!}$ and $\frac{n!}{r!(n-r)!}$ to show that they are the same.

23. Find the number of combinations of n elements taken r at a time.

24. Evaluate ${}_nC_r$ and ${}_nC_{n-r}$ to show that they are the same.

25. Define a sample space.

26. Define an event.

27. Define a probability.

28. Evaluate simple probability problems.

Objective	D_8	PA	D_3	PL	N
1	105	720	487	450	473
2	105	725	488	453	474
3	109	725	489	453	474
4	110	726	491	455	476
5	114	721	493	457	475
6	115	726	493	457	476
7	116	724	494	---	480
8	120	730	498	461	477
9	121	730	499	461	478
10	125	731	501	463	480
11	129	731	503	465	478
12	130	731	504	465	479
13	139	738	508	474	488
14	602	---	576	556	496

Objective	D_8	PA	D_3	PL	N
15	602	---	576	554	498
16	603	---	577	---	503
17	603	---	577	557	501
18	603	---	577	557	498
19	604	---	577	558	501
20	606	---	580	560	502
21	608	---	581	562	504
22	608	---	583	562	505
23	610	---	583	562	506
24	610	---	583	563	508
25	617	---	589	569	519
26	617	---	590	569	520
27	619	---	592	567	521
28	622	---	594	574	527

STRATEGIES

Objective

1 Demonstrate to students--given a few terms of a sequence it is not always possible to predict the next terms; good example to use is 3,5,7--the next term may be 9 or, if the sequence is that of prime numbers, it may be 11.

Another example: 5,7,9,--can be $2n + 3$ or $n^3 - 6n^2 + 13n - 3$.

3 Prove inductively $t_n = a + (n - 1) d$.

4 Show the arithmetic mean inserted between two numbers is the average.

6 Show the sum of the first n terms of an arithmetic progression can be represented by $S_n = a + (a + d) + \dots + [a + (n-1) d]$ x, then write the sum in reverse order using l for last term and add.

$$
\begin{aligned}
S_n &= a + (a + d) + (a + 2d) + \dots\ a + (n-1) d \\
S_n &= l + (l - d) + (l - 2d) + \dots\ l - (n-1) d \\
\hline
2S_n &= (a+l)+(a+l) + (a + l) + \dots + (a + l)
\end{aligned}
$$

$$2S_n = n\ (a + l)$$

$$S_n = \frac{n}{2}\ (a + l) \text{ from previous formula}$$

$$l = a + (n-1)\ d \text{ we have}$$

$$S_n = \frac{n}{2}\ [a + a + (n - 1)\ d] \text{ or } S_n = \frac{n}{2}\ [2a + (n-1)\ d]$$

9 Show $t_n = a\ r^{n-1}$ by chart. Use numbers first, then generalize. Let a represent the first term
r the common ratio
t_n the value of the nth term
n the number of the term

n	1	2	3	4	5	...	n
t_n	$a \cdot r^0$	$a \cdot r^1$	$a \cdot r^2$	$a \cdot r^3$	$a \cdot r^4$	...	$a \cdot r^{n-1}$

Objective

12 Show that the sum of the first n terms of a geometric series can be represented by

$$S_n = a + a\ r + a\ r^2 + a\ r^3 + \dots + a\ r^{n-2} + a\ r^{n-1}$$

Multiply each term of this equation by -r and add

$$S_n = a + ar + ar^2 + ar^3 + \dots a\ r^{n-2} + a\ r^{n-1}$$

$$\underline{-rS_n = -ar - ar^2 - ar^3 - ar^4 \dots - a\ r^{n-1} - a\ r^n}$$

$$S_n - rS_n = a - a\ r^n$$

$$(1 - r)\ S_n = a - a\ r^n$$

$$S_n = \frac{a - a\ r^n}{1 - r}$$

13 $$S_n = \frac{a\ (1 - r^n)}{1 - r} \quad \text{or} \quad \frac{a}{1 - r}\ (1 - r^n)$$

Show that r^n approaches zero in an infinite series, therefore the formula for infinite geometric series is:

$$S_n = \frac{a}{1-r}$$

15 Show by example. Take no more than four elements and write all permutations.

16 Circular permutations can be vividly demonstrated using three people. Show linear permutation of three people, then show circular permutation of three people.

24 Have students work out examples such as 20^C18 and 20^C2, then prove

$n^Cr = n^C\ n\text{-}r$ by using the formula $n^Cr = \frac{n!}{(n-r)!\ r!}$

PRETEST

1. Solve for the missing term.

$t_n = a + (n - 1)d$

a) $a = 1,\ d = 3,\ n = 10$
b) $t_n = 142,\ a = 2,\ d = 7$

2. Solve for the missing term.

$S_n = \frac{n}{2}\left[2a + (n-1)\ d\right]$

a) $a = \frac{1}{5},\ d = \frac{2}{5},\ n = 16$
b) $Sn = 275,\ d = 5,\ n = 11$

3. Solve for the missing term.

$t_n = a\ r^{n-1}$

a) $a = -9,\ r = 2,\ n = 4$
b) $t_n = 162,\ r = -3,\ a = 2$

4. Solve for the missing term.

$S_n = \frac{a - a\ r^n}{1 - r}$ or $\frac{a - rl}{1 - r}$

a) $a = 64,\ r = \frac{1}{4},\ l = \frac{1}{2}$
b) $S_n = -25,\ r = -2,\ a = 5$

5. Evaluate.

a) $_{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} =$

b) $_{60}C_3 = \frac{60 \cdot 59 \cdot 58}{1 \cdot 2 \cdot 3} =$

PRETEST ANSWERS

1. (a) $28 = t_n$

   (b) $21 = n$

2. (a) $51\frac{1}{5} = S_n$

   (b) $0 = a$

3. (a) $-72 = t_n$

   (b) $5 = n$

4. (a) $85\frac{1}{6} = S_n$

   (b) $-40 = l$

5. (a) 210

   (b) 34220

POSTTEST

OBJECTIVE	1-4 True or False
1	1. The following is a sequence $(1,a_1)$, $(2, a_2)$, $(3, a_3)$, $(4, a_4)$ . . . , (n, a_n)
1	2. A series is a function whose domain is the set of positive integers. The numbers contained in the range of the function are the terms of the series.
2	3. A sequence is the indicated sum of the terms in a series.
3	4. 1, 2, 3, 4, 5 is an arithmetic progression.
4	5. Find the nth term of the A. P. when $a = 2$, $d = 4$, and $n = 17$.
4	6. Which term of 18, 14, 10, . . . is -50?
5	7. Find the three arithmetic means between 2 and 14.
5	8. Find the arithmetic mean (average) of 0 and 3.
6	9. A ______ is the indicated sum of the terms in a sequence.
7	10. Find the sum of an arithmetic progression given $a = 5$, $n = 20$, and $l = 100$.
7	11. Find the sum of an arithmetic progression given $a = 2$, $n = 20$, and $d = 3$.
8	12. Find the sum of the arithmetic series $\sum_{n=1}^{5} 4n$
9	13. A geometric sequence is one in which the ______ of any term to its predessor is always the same number.
10	14. Find the eighth term of 4, 8, 16, 32 . . .
10	15. Which term of -1, -2, -4, . . . is -128?
11	16. Find the positive geometric mean of 5 and 45.
11	17. Insert three real number geometric means between 3 and $\frac{3}{16}$.
12	18. The sum of the terms in a geometric progression is a _____ __________.

13 19. Find the sum of a geometric series whose first term is 4, whose last term is 324, and whose common ratio is 3.

13 20. Find the sum of the first five terms of the geometric sequence 2, -8, +32, . . .

14 21. Change $0.\overline{12}$ to an equivalent common fraction using the formula for finding the sum of an infinite geometric progression.

15 22. A __________ is any arrangement of the elements of a set in a definite order.

16 23. In how many ways can you arrange 4 different books on a shelf?

17 24. In how many ways can four people be seated around a table?

18 25. $5^{P}3 =$

19 26. $5^{P}5 =$

20 27. In how many ways can you arrange 5 books on a book shelf that holds 3 books?

21 28. How many different permutations can be made from the letters of the word M I S S I S S I P P I?

22 29. A _________ is an arrangement of the elements of a set without consideration of the order of the elements.

23 30. Evaluate $\frac{10^{P}4}{4\,!}$ and $\frac{10!}{4!\,(10-4)\,!}$

24 31. In how many ways may a committee of 3 be chosen from a class of 30 students?

25 32. Evaluate $50^{C}\,2$ and $50^{C}\,48$.

26 33. A _________ is a set of S of elements that correspond one-to-one with the outcomes of an experiment.

27 34. An event is any ________ of a sample space.

28 35. A __________ is a number between 0 and 1 used as a mathematical model of the ratio of a particular outcome to the total number of outcomes in an experiment that is repeated or performed with a number of objects.

29 36. What are the odds that the drawing of a card at random from a deck of bridge cards will produce a king?

ANSWERS TO POSTTEST

1. True
2. False
3. False
4. True
5. 66
6. 18
7. 5, 8, 11
8. $1\frac{1}{2}$
9. series
10. 1050
11. 610
12. 60
13. ratio
14. 512
15. 8th term
16. 15
17. $-\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}$
18. geometric series
19. 484
20. 410
21. $\frac{4}{33}$
22. permutation
23. 5! or 120
24. (4-1)! or 6
25. 60
26. 5! or 120
27. 60
28. 3480
29. combination
30. $\frac{{}_{10}P_4}{4!} = 210$

    $\frac{10}{4!(10-4)!} = 210$
31. ${}_{30}C_3 = 4060$
32. ${}_{50}C_2 = {}_{50}C_{48} = 1225$
33. sample space
34. subset
35. probability
36. $\frac{1}{13}$